33B Midterm 2

Vedant Sahu

TOTAL POINTS

37 / 40

QUESTION 1

auto. Equation 11 pts

1.1 Phase Line 3 / 3

✓ **- 0 pts** Correct

- 1 pts arrows missing/wrong

- 1 pts graph wrong somewhere

- 1 pts missing scale

- 3 pts not the phase line

- 1 pts too many zero's in the graph

1.2 Eq. Solutions 3 / 3

✓ **- 0 pts** Correct

- 1 pts -3 no conclusion

- 1 pts 2 is stable

- 1 pts 5 is unstable

- 2 pts Stable/unstable?

1.3 Graph sketch 2 / 2

✓ **- 0 pts** Correct

- 2 pts no solution/wrong solution

- 1 pts graph between -3 and 2 wrong/missing

- 1 pts graph between 2 and 5 wrong/missing

1.4 particular solution 3 / 3

✓ **+ 3 pts** Correct

+ 1 pts No

+ 1 pts Uniqueness theorem can be applied

+ 0 pts wrong/no answer

+ 1 pts cannot cross the equilibrium solution y(t) = 2

QUESTION 2

Existence and Uniqueness 8 pts

2.1 Apply? Rectangle? 5 / 5

✓ **+ 2 pts** continuous

✓ **+ 2 pts** derivative continuous

✓ **+ 1 pts** rectangle

+ 0 pts no points

2.2 x_0(2)=5? 0 / 3

+ 1 pts Correct

+ 2 pts justification

✓ **+ 0 pts** no points

QUESTION 3

3 Particular Solution 6 / 6

✓ **- 0 pts** Correct

- 1 pts Mixed up a minus sign

- 3 pts Didn't try the right guess (ae^3t)

- 6 pts Didn't attempt method of undetermined coefficients.

- 1 pts Incorrect arithmetic in finding constant.

- 1 pts Incorrect multiplication

- 1 pts Put constant in solution

- 3 pts Forgot to include an undetermined coefficients in MOC.

QUESTION 4

2. order equation constant coefficients 5 pts

4.1 verify solutions 3 / 3

✓ **- 0 pts** Correct

- 2 pts Didn't explicitly check boundary conditions

- 1 pts Only checked one boundary condition

- 1 pts Didn't correctly check that they satisfy the ODE.

4.2 existence and uniqueness? 2 / 2

✓ **- 0 pts** Correct

- 2 pts Didn't understand that solution was non-unique.

- 2 pts Didn't state that the boundary conditions being defined at different times renders the existence and uniqueness theorem irrelevant.

- 1 pts Not clear if you actually meant that the

"initial" conditions are defined at different times.

QUESTION 5

2. order equation 7 pts

5.1 verify solutions **4 / 4**

- ✓ **- 0 pts** **Correct**
- **- 2 pts** incorrect calculation
- **- 4 pts** incorrect calculation

5.2 fundamental set **3 / 3**

- ✓ **- 0 pts** **Correct**
- **- 1 pts** conclusion is incorrect,
- **- 1 pts** some work, calculation incorrect,
- **- 3 pts** conclusion incorrect, wrong calculation
- **- 2 pts** some work

QUESTION 6

6 planar system **3 / 3**

- ✓ **- 0 pts** **Correct**
- **- 2 pts** incorrect, but some work
- **- 1 pts** minor mistake
- **- 3 pts** no work

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MIDTERM 2

11/16/2018

Math33B
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section: 2B

Problem	Points	Score
1	11	
2	8	
3	6	
4	5	
5	7	
6	3	
Total	40	

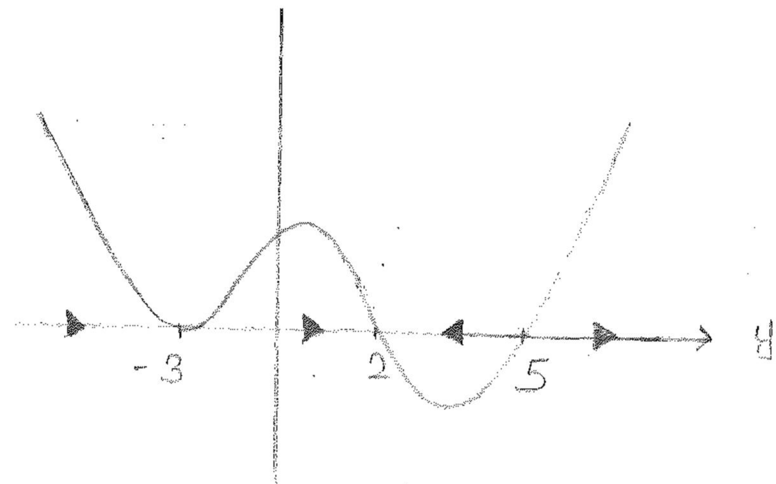
Instructions

(1) This exam has 6 problems. Make sure you have all pages.
(2) Enter your name, SID number, and discussion section on the top of this page.
(3) **Use a PEN** to record your final answers.
(4) If you need **more space**, use the extra page at the end of the exam.
(5) NO Calculators, computers, books or notes of any kind are allowed.
(6) Show your work. Unsupported answers will not receive full credit.
(7) Good Luck!

Exercise 1. (11pt)
Consider the autonomous first-order differential equation.

$$y' = (y+3)^2(y-2)(y-5)$$

(1) Draw the phase line. (3pt)



(2) What are the equilibrium solutions? Which are stable, and which are unstable? (3pt)

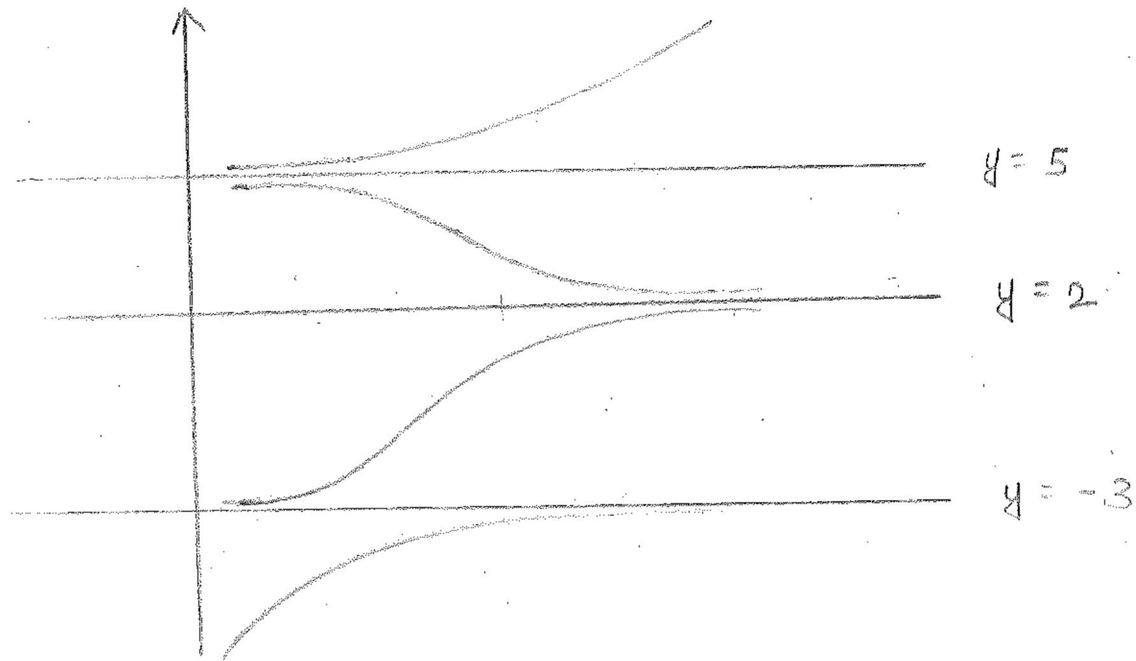
The ~~E~~ ~~equils~~ equilibrium solutions are :

$y = -3$ (Neither)

$y = 2$ (Stable)

$y = 5$ (Unstable)

(3) Sketch the graph of at least one solutions between each pair of adjacent equilibrium solutions. (2pt)



(4) Let $y_p(t)$ be a particular solution to the equation which satisfies $y_p(0) = 0$. Is it possible that $y_p(2) = 2$? Justify your answer. (3pt)

$f(t,y) = (y+3)^2(y-2)(y-5)$

$f(t,y)$ and $\partial f/\partial y$ are continuous on the entire ty-plane. Thus, uniqueness and existence theorem applies.

Now, $f(t,2)$ is a solution ~~for~~ to the equation for every t. So, if $y_P(2) = 2$ then ~~y_P~~ $y_P(0)$ must be equal to 2 due to the uniqueness theorem.

Therefore, if $y_P(0) = 0$, then it is not possible that $y_P(2) = 2$.

Exercise 2. (8pt) Consider the differential equation

$$\frac{dx}{dt} = \frac{\sqrt{x^2 - 4}}{t^2}.$$

(1) Can you apply the existence and uniqueness theorem to the initial value problem $x_0(1) = 6$? Justify your answer and give the biggest rectangle in which you can apply it containing the given point (if it exists). (5 pt)

$$f(x,t) = \frac{\sqrt{x^2-4}}{t^2}$$

This is continuous for all $t \neq 0$ and for all $x \notin (-2,2)$

$$\cancel{\frac{\partial f}{\partial t}} = \cancel{\frac{x}{\sqrt{x^2-4}\;t^2}} \quad \frac{x}{t^2\sqrt{x^2-4}} = \frac{\partial f}{\partial x}$$

This is continuous for all $t \neq 0$ and for all $x \notin [-2,2]$

There exist a rectangle R containing (1,6) for which both f and $\partial f/\partial x$ are continuous. Therefore, we can apply the uniqueness and existence theorem to the given initial value problem.

The biggest rectangle $R = \underbrace{(0,\infty)}_{t} \times \underbrace{(2,\infty)}_{x}$

(2) Can $x_0(2) = 5$ ($x_0(t)$ is the solution to the initial value problem in part 1))?(1pt) Justify your answer. (2pt)

In the rectangle R, ~~ff~~ there is no other solution with which $X_0(t)$ intersects.

Yes, ~~xt~~ $X_0(2) = \cancel{5}$ can be equal to 5.

$f(2,t)$ is a solution for all t but $X_0(t)$ does not intersect with $f(2,t)$ so it is entirely possible for an $X_0(t)$ to exist such that $X_0(1) = 6$ and $X_0(2) = 5$.

The only way $X_0(t)$ will not be a solution is if $X_0(t_1) = 2$ for some $t_1 \in (0,\infty)$. Otherwise, $X_0(t)$ can exist without contradicting the existence and uniqueness theorem.

Exercise 3. (6pt) Find a **particular** solution to the following differential equation

$$3y'' + 2y' - y = -4e^{3t}.$$

Let $y = ae^{3t}$

$y' = 3ae^{3t}$ $\quad y'' = 9ae^{3t}$

Now plugging it in

$3y'' + 2y' - y$

$= 27ae^{3t} + 6ae^{3t} - ae^{3t}$

$= 32ae^{3t} = -4e^{3t}$

$\Rightarrow a = -4/32 = -1/8$

Therefore, $y_P(t) = -\frac{1}{8}e^{3t}$

Exercise 4. (5pt) Consider the following problem:

$$y'' + y = 0 \qquad y(0) = 0 \;\; y'(\pi/2) = 0$$

(1) Show that $y(t) = C \cdot \sin(t)$ is a solution for any constant C. (3pt)

Characteristic equation : $\lambda^2 + 1 = 0$

$\Rightarrow \lambda = \pm i$

$a = 0$, $b = 1$

So, the 2 solutions to the homogenous equation are : $y_1 = C_1 \cos t$, $y_2 = C_2 \sin(t)$

Verifying :

$y(t) = C \cdot \sin(t)$

$y'(t) = C \cdot \cos(t)$, $y''(t) = -C \cdot \sin(t)$

$y'' + y = -C \sin(t) + C ~~\sin t~~ \sin(t) = 0$

for any constant C

Also, $y(0) = C \sin(0) = 0$

~~$y'(0) = C \cos$~~ $y'(\pi/2) = C \cos(\pi/2) = 0$

Hence, verified

(2) Why does this not violate the 2. order existence and uniqueness theorem? (2pt)

The 2 order existence and uniqueness theorem requires y_1 and y_2 such that $y_1 = y(t_0)$ and $y_2 = y'(t_0)$. Here the value of y_1 and y_2 given are not for the same $t = t_0$. ~~$0 = y(t_0)$ when~~ $y_1 = y(0)$ whereas $y_2 = y'(\pi/2)$. That is why the 2 order existence and uniqueness theorem is not ~~viole~~ violated.

For this solution, $y'(0) = C$ which will give ~~a~~ different values for different constants C, thus complying by the existence and uniqueness theorem.

Exercise 5. (8pt) Consider the differential equation

$$y'' + \frac{1+x}{x}y' - \frac{1}{x}y = 0$$

(1) Check that $1+x$ and $\frac{2x^2+6x+4}{x+2}$ are solutions to the above equation. (4pt)

Let $y_1 = 1+x$

$y_1' = 1 \quad , \quad y_1'' = 0$

$$y_1'' + \frac{1+x}{x} y_1' - \frac{1}{x} y_1$$

$$= 0 + \frac{1+x}{x} - \frac{1}{x}(1+x) = 0$$

$\therefore \; y_1 = 1+x$ is a solution

Let $y_2 = \frac{2x^2+6x+4}{x+2} = \frac{2(x^2+3x+2)}{x+2}$

$$= \frac{2(x+1)(x+2)}{x+2}$$

$$\underset{x \neq -2}{=} 2(x+1)$$

$y_2 = 2y_1$

We know that y_1 is a solution

So $y_2 = 2y_1$ must also be a solution to the above equation.

Therefore, both $1+x$ and $\frac{2x^2+6x+4}{x+2}$ are solutions.

(2) Do they form a fundamental set of solutions?(1pt) Justify your answer. (2pt)

No, they do not form a fundamental set of solutions.

$y_2 = 2y_1$

So y_1 and y_2 are linearly dependent

But, for them to form a fundamental set of solutions, they need to be linearly independent, which is not true.

Exercise 6. (3pt)
Consider the second order equation

$$y'' - 2e^t y' - \tan(t)y = \sqrt{t^2+1}.$$

Write this equations as a planar system of first-order equations.

Let ~~$y' =$~~ $x = y'$

Then ~~x~~

$$\left.\begin{aligned} y' &= x \\ x' &= 2e^t x + \tan(t)\, y + \sqrt{t^2+1} \end{aligned}\right\} \text{Planar system}$$

Extra page

Extra page

Rough work

$f(t, y) = C \sin t$ } cont.

$\frac{\partial f}{\partial y} = 0$ } cont.